

Christmas puzzle 2021



Solution booklet

Dear puzzler

Perhaps I should start with a small apology. In my zeal to make the 2021 Christmas card design as beautiful as possible, I may have been a little too sober in the explanation department.

It was probably obvious that the design on the inside is an encoded text. It was probably not obvious that the design on the front was intended as a tool to help you decode it. I should have said that explicitly. After working on it for quite some time, it simply never occurred to me that it might not be obvious.

With the previous card working exactly opposite (you solved the inside to figure out what the outside meant) this was quite an unfortunate oversight on my part.

So, for that, I apologize.

I received feedback from several solvers leading me to try something new with the solution booklet. I've tried to set things up so that you can read one section at a time, and whenever you think you have seen the clue that makes it click you can then try figuring it out yourself from there.

Step one. Examining the puzzle for clues

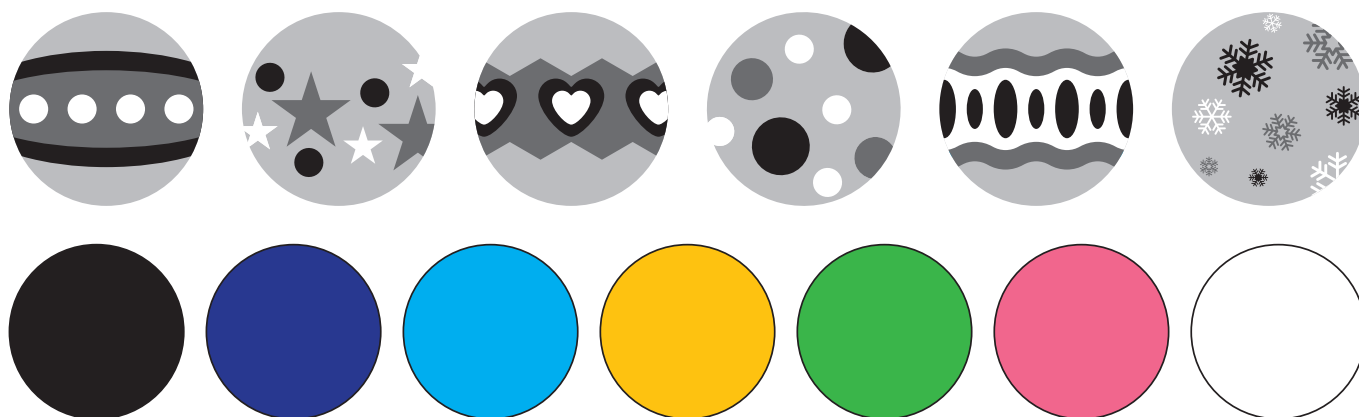
My favourite part of puzzling is the 'break in' - finding the starting point. We need to figure out what exactly we are dealing with. The following facts are (hopefully) not too hard to see:

- No two balls on the front are exactly the same. There are six different patterns and seven different colours, combined in a variety of ways.
- The balls on the inside do repeat, but none of them occur exactly in that form on the front.
- The balls in either design have these pointy bits attached; on the inside they seem to group the balls, on the front they appear to be random.

The question that I hoped would come to you at this point is: so what, exactly, does link the two designs?

About the patterns and colours

It was vital to find seven colours that contrast well with each other and the background red, so when the cards came from the printer I was quite dismayed to see the dark blue came out too close to the black. The patterns also had to be easy to discern on the small scale, but I think that worked out well in the end.



Step two. Number theory

The object is to find something, anything, that repeats or reoccurs on the front. If we are to find clues for the inside there, there MUST be something.

If you examine the front it should become apparent that there is no rhyme or reason to the occurrence of each ball pattern. Since there are only six patterns, there is just not a lot of information there: the ball patterns repeat, a lot, but not meaningfully.

There are a lot more options for the colours, since we can also vary the number of colours on each ball. So it makes sense to focus on the colours first. Are all options also used?

In fact, is there something you can deduce about the numbers of colours used?

About the maths of choosing

Say we divide a ball into three sections to be coloured. How many different ways could it be coloured in with the same three colours? We have 3 options for the first, two remaining for the second, and whatever we haven't used yet is our third: $3 \times 2 \times 1 = 6$ possible ways. This is called the factorial $N!$ of a number N .

There are seven ways to pick one colour out of seven options (obviously). How many ways are there to pick two colours? Well, you have seven choices for the first, then six remain for the second. BUT. Now you treat 'pink, green' and 'green, pink' as different combinations and that's a bit silly.

So there are really only $7 \times 6 / 2! = 21$ possible ways.

In the same way there are $7 \times 6 \times 5 / 3! = 35$ ways to pick three colours. If you pick three, you leave four of them unpicked, so every way to pick three is also a way to pick four - there are 35 ways to pick four colours. Similarly, there are 21 ways to pick five, and seven ways to pick six; there is only one way to pick all seven or none at all.

(None of this, by the way, is relevant to the puzzle.)

Step three. Colour theory

Ok. So every ball on the inside uses a different combination of four out of the seven colours. What about the front?

There's a lot more balls to check on the front, but the large majority of them *also* use four colours. There are only 35 ways of picking four colours out of a set of seven, so this means some colour combinations **MUST** occur more than once on the front. On average, it would be about four times per colour combination.

So what happens if we pick a colour combination and try and find all balls that use it on the front?

Below you can see the front of the card, but with all balls that have the same colours as the top left one, pink/white/green/light blue (in some combination), replaced with a 'bullseye' ball. I've dimmed the others to make them stand out:



Step four. Connect the dots

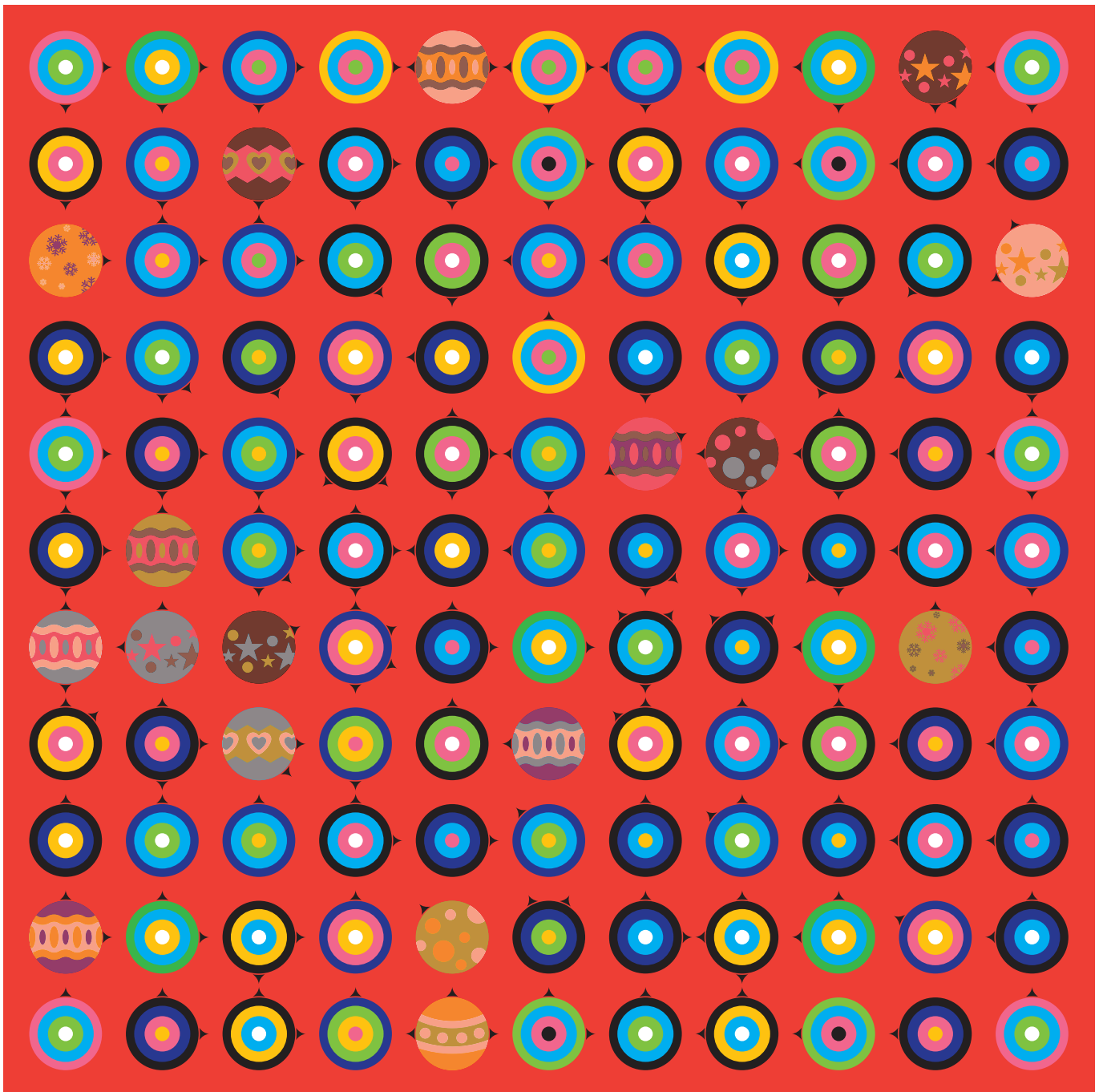
Now the role of the little black pointy bits becomes clear as well (or so I hope).

In the previous illustration, the bullseye balls point at each other. They form a (crude) A if you isolate the pattern and squeeze your eyes a bit.

It's not easy to spot the balls with the same colours, because it makes a real difference in how they are perceived depending on whether they are used on details or large surfaces. But that is all part of the deliberate obfuscation.

Once you see the role of the black connectors, though, you can pick a ball, follow the direction of the pointy bit and pretty quickly pick out a matching ball.

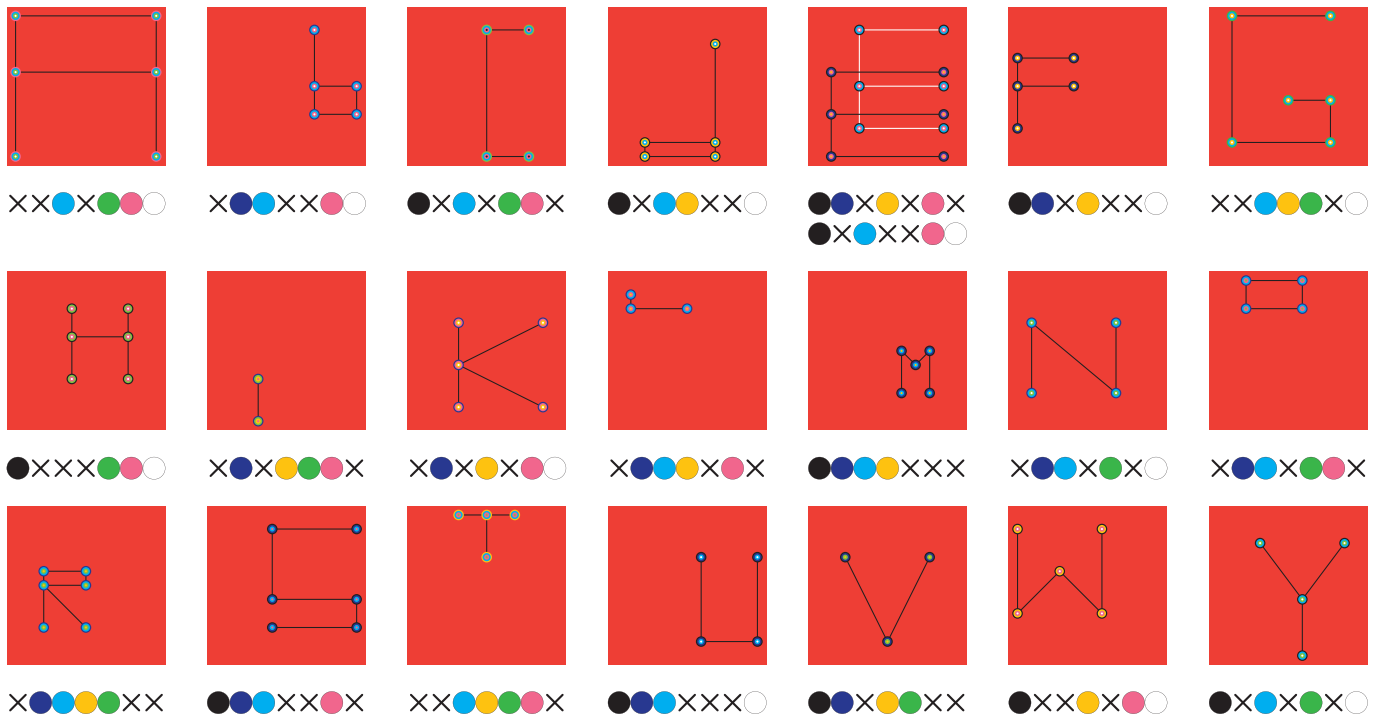
This way it should be doable to find all letters hidden in the front. The other 17 balls (shown dimmer here) are red herrings.



Step five. Crack the code

It's now a simple matter of matching the colours in the encoded text to the colours in the hidden letters, and reconstruct the text.

Here's an overview (there is a G in the grid, but it is not used):



And here is the encoded text reduced to its colours, with the solution:



Conclusion

The original concept had a lot more balls and used multiple patterns for each letter (like the E in the final version). This made it next to impossible to decode the text directly. In order to keep things readable on the Christmas card format, I had to reduce the number of balls to make them large enough; the end result is at the absolute limit of what I could squeeze out.

I guess that it's a good thing in the end, because it opened up an alternative path to the solution. (Although that E may have thrown a few people, or so I hope).

If I could do this one again, I would probably use a seventh unique pattern for the balls used in the encoded text. It would point you more directly towards the colours. I'd also give you just a tad more instructions.

Well we live and learn. Number four is bound to be perfect...

I always tell people that a smart puzzle takes longer to solve than to make, but this is rarely the case. And in retrospect I gave myself way too many fiddly little tasks to perform. Despite my best efforts, two errors slipped through - two balls on the front had too many 'pointy bits'. I've corrected this in this solution booklet, let's call it a little extra puzzle to identify them.

It probably didn't affect the difficulty, because if you got far enough that you were constructing the letters, this should have been auto-correctable.

Would you believe it's almost time to start the next card! The next one will be... more straightforward again. Well, that's the plan anyway.

As always, do not hesitate to send me feedback - I promise you won't hurt my feelings if you give me pointers to improve things.